ION-SOUND WAVE STRUCTURE IN A POINT EXPLOSION

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Ionized regions develop ahead of shock waves for various reasons [1-5]. It is known [6] that in shock wave propagation in a plasma, electron thermal conductivity produces a heating layer ahead of the front with characteristic spatial dimension $\Delta_T \approx [m_i/m_e]^{1/2}\ell$, where $T_e > T_i$ [subscripts e and i indicating electrons and ions (the presence of ions of only one type is assumed), m is mass, T, temperature, and ℓ, the ion mean free path length]. Immediately behind the wave front an abrupt increase in ion temperature occurs, while electron temperature remains practically unchanged, after which T_e and T_i slowly equalize in the relaxation region. In principle the temperatures in question behave in the same manner in shock wave propagation in an initially non-isothermal plasma [7]. Ahead of the shock fronts the effect of preceding phenomena asserts itself. Avramenko et al. [8] considered a mechanism to explain increase in the degree of plasma ionization ahead of a viscous compression discontinuity in the absence of ionizing radiation from the wave front. This is due to formation of an ion-sound shock wave. The steady state problem of shock wave structure in a weakly ionized plasma was solved [8] under the condition that a planar shock wave be specified within the neutral component. The proposed approach can be generalized to the non-steady case, where the neutral component moves as a result of the action of a finite energy source E_0 by a specified law. In particular, we may consider as an idealized situation the self-similar motion of the neutral component under the action of a point energy source [9].

We will find the perturbation to the density of the plasma component in the heating layer, assuming first of all that the spatial scale ξ of the density change is small in comparison to the width of the heating layer. In as much as $\xi \approx (V_s/c)^2\ell$, where $V_s = [zk_B(T_e + T_i)/m_i]^{1/2}$ is the speed of ion-sound, $z = |e_i/e| = 1$ is the charge number, k_B is Boltzmann's constant, and c is the velocity of the viscous compression discontinuity, we have a limitation imposed upon the shock wave intensity:

$$(V_s/c)^2 \ll (m_i/m_e)^{1/2}.$$

In the heating layer we write the plasma dynamics equations in the form

$$\partial(r^{j-1}\rho)/\partial t + \partial(r^{j-1}\rho v)/\partial r = 0; \quad (1)$$

$$\partial v/\partial t + v\partial v/\partial r = -(V_s^2/\rho)\,\partial\rho/\partial r - \nu_{in}(v - v_n). \quad (2)$$

Here j = 1, 2, 3 for planar, cylindrical, and spherical waves; ρ and v are the mass density and velocity of the plasma component; r and t are the coordinate and time. The velocity of the neutral component $v_n(r, t)$ will be considered known (from the point explosion problem of [9]). We neglect any reciprocal action of the charged component upon the neutral, since $\rho_0/\rho_{n0} \ll 1$ (weakly ionized plasma), ρ_n is the mass density of neutrals, and the subscript 0 denotes the undisturbed state of the fields. In essence, Eqs. (1), (2) describe the dynamics of the ion-sound wave excited by the outside source $v_n(r, t)$.

The self-similar motion of the neutral component is described by the dimensionless variable $\lambda = r/r_2(t)$, where $r_2(t) = [E_0\varkappa^{-1}(\gamma)\rho_{n0}^{-1}]^{1/(2+j)}t^{j/(2+j)}$ is the law of motion for the front; the subscript 2 denotes fields on the front; $\varkappa(\gamma)$ is a dimensionless coefficient of the order of magnitude of unity [9]. The front velocity is

$$c = \frac{2}{2+j}\left(\frac{E_0}{\varkappa\rho_{n0}}\right)^{1/(2+j)} t^{-j/(2+j)} = \frac{2}{2+j}\frac{r_2(t)}{t}.$$

For a strong shock wave, on the front we have the conditions

$$v_{n2} = 2c(\gamma+1)^{-1}, \quad \rho_{n2} = \rho_{n0}(\gamma+1)(\gamma-1)^{-1}$$

(γ is the adiabatic index for the neutral gas).

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Quasi-steady Approximation. We introduce a coordinate system attached to the front:

$$t' = t, \quad r' = r - \int_0^t c(\tau)\, d\tau.$$

In the vicinity of the front the particle velocity v is defined by the velocity c, and the explicit dependence upon t' is slow. Therefore the local algebraic relationship (polarization ratio) between ρ and v can be found approximately from the condition that the fields are quasi-static in the primed coordinate system. Neglecting the term containing $\partial/\partial t'$ in Eq. (1) we have:

$$\rho(r', t') \approx K(t') \{r^{j-1} [c(t') - v(r', t')]\}^{-1}. \tag{3}$$

Here K(t') is an integration constant, which can be expressed in terms of the field values on the front:

$$K(t') = \rho_2 c r_2^{j-1} \rho_{n0}/\rho_{n2}. \tag{4}$$

In the ion-sound wave problem of [8] the inequality $V_s^2 \gg cv$ is satisfied, so that Eqs. (2), (3) can be solved approximately, if we neglect the terms with dv/dt in Eq. (2). As a result we obtain an ordinary differential equation for $u = [(c - v)r^{j-1}]^{-1}$ (the variable t appearing as a parameter):

$$du/dr - u(v_n - c)\, \nu_{in}/V_s^2 - \nu_{in}/(V_s^2 r^{j-1}) = 0. \tag{5}$$

In Eq. (5) $\nu_{in}(r)$ and $v_n(r)$ are discontinuous functions at $r = r_2(t)$. In the region ahead of the front at $v_n = 0$, $\nu_{in} = \nu_{in}^0$, Eq. (5) has the general solution

$$u = u(r_2) \exp[-(r - r_2)/\xi] + \Phi(r), \tag{6}$$

where $\xi = V_s^2/[c(t)\, \nu_{in}^0]$; $\Phi(r) = \frac{\exp(-r/\xi)}{c\xi} \int_{r_2}^{r} dx\, x^{1-j} \exp(x/\xi)$.

For $j \neq 1$ the variable Φ can be written in a form asymptotic in the parameter $\alpha = \xi(t)/r_2(t) < 1$, while for v we write approximately

$$v(r, t) \cong c(1 - R_j)\{1 - R_j + \exp[(r - r_2)/\xi - \mu]\}^{-1}, \quad r > r_2(t). \tag{7}$$

Here

$$\mu = \ln[(r/r_2)^{j-1} v_2 (c - v_2)^{-1}], \quad R_j \approx \frac{c - v_2}{v_2} \left\{ \sum_k (k + j - 2)!\alpha^k - \left(\frac{r_2}{r}\right)^{j-1} e^{(r-r_2)/\xi} \sum_k (k + j - 2)!\alpha^k \left(\frac{r_2}{r}\right)^{j-1} \right\}, \quad j > 1.$$

For j = 1 Eq. (7) is an exact representation of Eq. (6), therefore $R_1 \equiv 0$. For the density ρ' of the plasma component ahead of the front we have

$$\rho(r, t) \cong \rho_2 \frac{\rho_{n0}}{\rho_{n2}} \left((1 - R_j) \frac{v_2}{(c - v_2)} e^{-(r-r_2)/\xi} + \left(\frac{r_2}{r}\right)^{j-1} \right), \quad r > r_2(t). \tag{8}$$

We must call attention to the unsatisfaction of the term "ion-sound shock wave" proposed in [8]. As is well known, an increase in the dissipative coefficient should lead to increase in thickness of the wave front. In the current case we have the reverse dependence: with increase in ν_{in}^0 the thickness ξ decreases.

Behind the front [at $r < r_2(t)$] we can obtain an approximate expression for the fields by taking $v \cong v_n(r, t)$:

$$\rho(r, t) \cong \rho_2 \rho_{n0} \rho_{n2}^{-1} c (c - v_n)^{-1} (r_2/r)^{j-1}, \quad r < r_2(t). \tag{9}$$

Then $v \cong v_{n2}$ and from the relationships upon the front it follows that $(c - v_2)/v_2 = \rho_{n0}/(\rho_{n2} - \rho_{n0}) \equiv (\gamma - 1)/2 = \text{const}$.

If we assume that as in the planar steady state problem of [8] for ρ_2 we have the condition

$$\rho_2 = \rho_0 \rho_{n2}/\rho_{n0} = \rho_0 (\gamma + 1)/(\gamma - 1), \tag{10}$$

then for j = 1 Eqs. (7)-(9) coincide formally with those found in [8]. For $j \neq 1$ a factor appears describing the divergence of $(r_2/r)^{j-1}$. In this case the disturbance of the plasma component drives the neutral wave front further, the greater j. In the non-steady case under consideration here a relationship like Eq. (10) must be found from the condition of conservation of the finite energy liberated E_0. With consideration of this, the solution of Eqs.

(7)-(9) can be considered as a generalization of the solution of [8] to the non-steady problem of one-dimensional motion of the plasma component near the front as a result of point energy liberation.

The non-steady nature of the wave of Eqs. (7), (8) follows from the fact that its form is defined by the self-similar variable $\lambda = r/r_2$, the front width $\xi(t)$ increasing with time, while the amplitude of the velocity decreases as $c(t)$. The total energy H of the plasma component included in a "sphere" of radius r, at the time t has the form

$$H(r, t) = 2^{j-1}\pi_j \int_0^r dx x^{j-1} \rho \left(V_s^2/(\gamma_p - 1) + v^2/2\right), \tag{11}$$

where γ_p is the adiabatic index for the plasma component, $\pi_j \cong 3.141$ for j = 2, 3 and 1 for j = 1. We neglect the contribution to H of the electric field with density equal to $D^2(\nabla\rho/\rho)^2\rho_0 V_S^2/2$ (where D is the Debye screening radius) in the long-wave description of Eqs. (1), (2). Then the boundary condition with consideration of loss of ion mechanical energy due to collisions with the neutral component (we neglect losses due to electron friction) can be written as

$$[H(r, t) - H_0(r) - \Omega(r, t)]_{r\to\infty} = E_p,$$
$$\Omega(r, t) = 2^{j-1}\pi_j \nu_{in}^0 \rho_0 \left\{ \int_0^{r_1} dx x^{j-1} \int_x^{r_2(\theta)} ds v(s) + \int_{r_1}^{r} dx x^{j-1} \int_x^{s(t,x)} ds v(s) \right\}, \tag{12}$$
$$\Omega(r, t) = \Omega_1(r) + \Omega_2(r, t).$$

Here $H_0(r, t) = H(r, t)$ for $t < 0$; E_p is the portion of the liberated energy E_0 transferred to the plasma component for an explosion at the time t = 0. If E_0 is uniformly distributed over the components, then $E_p \cong E_0\rho_0/\rho_{n0}$. The internal integrals in Eq. (12) represent the work of friction forces along the trajectories of motion s(t, x) of a "material point," which is located at the point x at the initial moment t = 0. The term Ω_1 considers the contribution of material points which are found behind the shock wave front by time t. The friction force then performs work only along the portions of the trajectories located ahead of the shock front, since in our model complete entrainment of the plasma occurs behind the front, $v = v_n$. It follows from this that $r_1(t)$ is a solution of the equation $s(t, r_1) = r_2(t)$, while the time $\theta(x)$ at which the shock wave overtakes the "material point" which was located at a distance x from the center at t = 0 is determined by the equation

$$s(\theta, x) = r_2(\theta). \tag{13}$$

The term Ω_2 reflects the contribution of points with $x > r_1(t)$, while the work is calculated along the segment included between x and s(t, x).

The trajectory of the motion of a "material point" s(t, x) is described by the equation

$$ds/dt = v(s(t), t) \tag{14}$$

with the initial condition s(0) = x and is known, if the solution of the problem v(r, t) is found in the region $r > r_2$ and r > 0. Making use of Eq. (14), in the internal integrals of Eq. (12) we can turn to integration over time, and the expression for $\Omega(r, t)$ takes the form

$$\Omega(r, t) = 2^{j-1}\pi_j \nu_{in}^0 \rho_0 \left\{ \int_{r_*}^{r_1} dx x^{j-1} \int_{x/V_{s1}^*}^{\theta(x)} dt' v^2(t') + \int_{r_1}^{r} dx x^{j-1} \int_{x/V_{s1}^*}^{s(t,x)} dt' v^2(t') \right\}, \tag{15}$$

where $V_{s1}^* \cong V_{s1}(0)$; $V_{s1}^2(t) \cong k_B(T_{i0} + T_{e2})/m_i$ is the velocity of ion-sound ahead of the shock wave front for t > 0; r_* is the minimum scale with which the initial stage of disturbance development can be excluded, when "material points" located ahead of the shock wave front are not set into motion. The value of r_* can be found by solving Eq. (13) for $\theta \cong r_2(\theta)/V_{s1}^*$.

To find ρ_2, as follows from Eqs. (11), (12), we must know the distributions of ρ, v, T_e, T_i in the region $0 < r < r_2(t)$. However near the center of symmetry it becomes impossible to neglect temperature gradients and the model of Eqs. (1), (2) is not suitable. We will make use of approximate expressions for the fields at $0 < r < r_2$: $v \cong v_n$, $T_i \cong T_n$, $T_i < T_e$ in the relaxation zone and $T_e \cong T_i$ outside that zone. Further, in order that the internal energy of the plasma component included inside the "sphere" of radius $r_2(t)$ remain finite, we require that $\rho/\rho_2 \approx \rho_n/\rho_{n2}$ as $r \to 0$ at any t. Thus, the values of ρ/ρ_2 and ρ_n/ρ_{n2} differ by less than an order of magnitude everywhere in the region $0 < r < r_2$. Then in the case of spherical symmetry

$$4\pi(\gamma_p-1)^{-1}\int_0^{r_2} drr^2\rho V_s^2 = \frac{4\pi}{\gamma_p-1}\frac{k_B}{m_i}\left\{2\int_0^{r_2} drr^2\rho T_n + \int_{r_2-\Delta_T}^{r_2} drr^2\rho(T_e-T_n)\right\} \cong$$

$$\cong 4\pi(\gamma_p-1)^{-1} r_2^3\rho_2 [2QBk_BT_{n2}/m_i + (\Delta_T/r_2)\varphi_1 V_{s0}^2].$$

Here $\varphi_1 = \varphi_1(\Delta_T/r_2)$ and Q is a factor of the order of unity; $B = \int_0^1 d\lambda\lambda^2 h(\lambda)$; $h(\lambda) = p_n/p_{n2}$ is a known function of the point explosion problem of [9]; p is the pressure. Using the expression for the temperature discontinuity upon the front of an intense shock wave $T_{n2} = T_{n0}(2\gamma/q)(\gamma-1)\times(\gamma+1)^{-2}$, $q = (a_0/c)^2$ (a_0 is the unperturbed speed of sound in the neutral gas, $a_0^2 = \gamma V_{s0}^2 T_{i0}/T_{e0}$), considering that $a_0^2 = \gamma k_B T_{n0}/m_i = \gamma p_{n0}/\rho_{n0}$, $E_p \cong E_0\rho_0/\rho_{n0}$, and neglecting losses ($\Omega \ll E_p$, and the effect of dissipation will be evaluated below), from Eqs. (11), (12) for j = 3 we find

$$\frac{\rho_2}{\rho_0}(\gamma-1)\left[A(\gamma)\frac{\gamma_p-1}{\gamma+1} + 2QB + \left(\frac{V_{s0}}{c}\right)^2\frac{\Delta_T}{r_2}\varphi_1\frac{(\gamma+1)^2}{2}\right] = \frac{(\gamma+1)^2}{2}\left[\left(\frac{V_{s0}}{c}\right)^2\frac{1}{3} + \frac{25\varkappa}{8\pi}(\gamma_p-1) - \frac{\Omega+\Psi'+\Psi}{4\pi r_2^3\rho_0 c^2}(\gamma_p-1)\right], \quad (16)$$

where

$$A(\gamma) = \int_0^1 d\lambda f^2(\lambda)\,[1-2f(\lambda)/(\gamma+1)]^{-1}; \quad A(1{,}4) \cong 0{,}7433; \quad B \cong 0{,}1588;$$

$$\Psi = 4\pi\int_{r_2}^{\infty} drr^2\,[(\rho-\rho_0)V_s^2/(\gamma_p-1) + \rho v^2/2];$$

$$\Psi' = 4\pi(\gamma_p-1)^{-1}\rho_0\int_{r_2}^{\infty} drr^2(V_s^2 - V_{s0}^2); \quad f(\lambda) = (v_n/c)(\gamma+1)/2.$$

Within the framework of the approximation used one cannot obtain Ψ as a function of ρ_2, since expressions for ρ and v are lacking for $r \gg r_2$. But the function Ψ in Eq. (16) yields a small contribution (to the estimated below), and to obtain an approximate expression for ρ_2 we can use the value Ψ = 0. Then, considering the term with Ψ', related to electron thermal conductivity, we obtain an amplitude value dependent upon the parameters V_{s0}/c and Δ_T/r_2:

$$\rho_2 \cong \rho_0\frac{\gamma+1}{\gamma-1}\,\frac{\frac{25\varkappa}{32\pi}(\gamma+1)^2(\gamma_p-1) - \frac{\Delta_T}{r_2}\varphi_2(\gamma-1) + \left(\frac{V_{s0}}{c}\right)^2\frac{(\gamma+1)^2}{2}\left(\frac{1}{3} - \frac{\Delta_T}{r_2}\varphi_2\right)}{(\gamma_p-1)A(\gamma) + (\gamma+1)\,[2QB + (\Delta_T/r_2)\varphi_1(V_{s0}/c)^2(\gamma+1)^2/2]}. \quad (17)$$

Here $\varphi_2 \equiv \varphi_2(\Delta_T/r_2) = 0.8956[1 + 0.9305\Delta_T/r_2 + (\Delta_T/r_2)^2 5/17]$ for the case of the T_e distribution typical of a thermal wave [6].

Solution Close to Self-similar. Another approximate representation of the solution of system (1), (2) with a wider range of applicability can be constructed. We do this in the dimensionless variables $\lambda = r/r_2$, $q = (a_0/c)^2$. We neglect the term containing $q(\partial/\partial q)$ in Eqs. (1), (2) since $q \ll 1$. Then, considering that the fields depend upon q as a parameter, we have

$$-c\lambda^j d\rho/d\lambda + d(\lambda^{j-1}\rho v)/d\lambda \cong 0; \quad (18)$$

$$-c\lambda dv/d\lambda + v dv/d\lambda + (V_s^2/\rho)\,d\rho/d\lambda \cong \nu_{in}(v_n - v). \quad (19)$$

From Eq. (18) we find

$$\rho^{-1}d\rho/d\lambda = [\lambda^{1-j}/(\lambda c - v)]\,d(\lambda^{j-1}v)/d\lambda. \quad (20)$$

Ahead of the wave front $v_n = 0$, so that after substitution of Eq. (20) into Eq. (19) we obtain an equation for $v(\lambda)$ at $\lambda > 1$, considering that $V_s^2 \gg vc$:

$$(v-\lambda c)^{-1}d(\lambda^{j-1}v)/d\lambda \cong \alpha^{-1}\lambda^{j-1}v/c.$$

The solution of this equation for the initial condition $v = v_2 \cong v_{n2}$ can be represented in the form

$$v = c\lambda^{j-1}\,[e^{(\lambda^2-1)/(2\alpha)}(\gamma+1)/2 - e^{\lambda^2/(2\alpha)}\int_1^{\lambda} dx x^{1-j}e^{-x^2/(2\alpha)}]^{-1}, \quad (21)$$

or, retaining the first term in the representation of the integral asymptotic in the parameter $\alpha \ll 1$ in Eq. (21), we obtain approximately

$$v \cong c\lambda\,\{1 + [(\gamma-1)/2]\,\lambda^j\exp[(\lambda^2-1)/(2\alpha)]\}^{-1}, \quad \lambda > 1. \quad (22)$$

For $\rho(\lambda)$, we find from Eqs. (20), (22)

$$\rho(\lambda) \simeq \rho_2 + (\rho_2 - \rho_0)\{\lambda^{-j} \exp[(1 - \lambda^2)/(2\alpha)] - 1\}, \quad \lambda > 1. \tag{23}$$

Comparing the result of Eqs. (22), (23) with the wave front structure of Eqs. (7), (8) we note that the stricter consideration of non-steadiness produces an increase in front curvature, which increases with increase in j. In fact, the quasi-steady approximation of Eq. (8) with the condition of Eq. (10) on the shock front yields

$$\rho \cong \rho_0 (\gamma - 1)^{-1} \{\gamma - 1 + 2 \exp[(1 - \lambda)/\alpha]\}, \quad \lambda > 1, \tag{24}$$

while the solution of Eq. (23), close to self-similar, when Eq. (10) is considered, yields

$$\rho \cong \rho_0 (\gamma - 1)^{-1} \{\gamma - 1 + 2\lambda^{-j} \exp[(1 - \lambda^2)/(2\alpha)]\}, \quad \lambda > 1. \tag{25}$$

For $\alpha = \xi/r_2 \ll j^{-1}$ Eq. (24) coincides approximately with Eq. (25) in the region $r_2 < r < r_2 + \xi$. In the quasi-steady approximation Eqs. (7), (8) represent a solution in the vicinity of the neutral wave front, and the approximations of Eqs. (22), (23) serve in the region $r_2 < r < r_2 + \Delta_T$, $\Delta_T \approx \ell(m_i/m_e)^{1/2}$. The approximate transformation of Eq. (23) into Eq. (8) near the wave front serves as justification of the assumptions made in deriving Eqs. (7), (8). The accuracy of the solution of Eqs. (7), (8) falls with increase in the order of symmetry. In this sense the quasi-steady approximation is also quasiplanar.

To derive Eq. (17) the simple relationships of Eqs. (3), (4), (9) of the quasi-steady approximation were used in place of Eq. (20) and $v \cong v_n$, $\lambda < 1$. This is possible because the main contribution to the integrals of Eq. (11) is made by disturbance of the plasma component near the shock wave front.

From Eqs. (16), (22), (23) with consideration of the fact that Ψ changes but little upon change of the upper integration limit in Eq. (16) from ∞ to Δ_T, we have

$$\Psi = 4\pi\alpha r_2^3 c^2 \{\rho_2 [(V_{s1}/c)^2 + \Gamma] - \rho_0 [(V_{s1}/c)^2 + \Gamma - \Gamma_1]\}, \tag{26}$$

where

$$\Gamma \cong (\gamma_p - 1)\{\gamma/(\gamma + 1) - (\gamma - 1) \ln[(\gamma + 1)/(\gamma - 1)]/2\};$$
$$\Gamma_1 \cong (\gamma_p - 1)\{(1/2) \ln[(\gamma + 1)/(\gamma - 1)] - (\gamma + 1)^{-1}\} > 0.$$

The presence in Eq. (26) of the factor $\alpha \ll 1$ indicates that for sufficiently narrow fronts $[\xi < r_2(t)(c/V_{s1})^2]$ the effect of spreading of the plasma disturbance upon the amplitude ρ_2 may be neglected. It also follows from Eqs. (16) and (26) that front broadening leads to a decrease in ρ_2.

We will now estimate the effect of dissipation of the mechanical energy of plasma component motion due to transfer of momentum to the neutral component by the "friction" path. Using Eq. (22), from Eq. (14) we obtain for the function $\sigma(t) = s(t)/r_2(t)$ the equation

$$d\sigma/dt = [j/(2 + j)] [\hat{v}(\sigma) - \sigma]/t, \quad \hat{v} \equiv v/c.$$

Integrating from t to $\theta(x)$, $t > r/V_{s1}^*$, to the accuracy of a term of the order of magnitude of α, we find $s(t) \cong x \cong r_2(\theta) \cong \text{const}$, $r_1(t) \cong r_2(t)$. In the same approximation for the rectified trajectories of Eq. (15) we have:

$$\Omega(r, t) \cong \frac{8\pi r_2^5}{5t} \rho_0 v_{in}^0 \left\{\int_{\Pi_1}^{1} dy\, y^{-5/2} \int_1^{\Pi_2(y,t)} d\lambda\, \lambda^{-1/2} \hat{v}^2(\lambda) + \int_1^{r/r_2} dy\, y^{-5/2} \int_y^{\Pi_2(y,t)} d\lambda\, \lambda^{-1/2} \hat{v}^2(\lambda)\right\}.$$

Here $y = x/r_2(t)$; $\Pi_1 = (5c/2V_{s1}^*)^{2/3}$; $\Pi_2(y, t) = x r_2^{-1} (x/V_{s1}^*)$. An upper limit estimate yields

$$\Omega \equiv \Omega(r, t)\big|_{r\to\infty} \leqslant (8\pi/5)(E_0\rho_0/\rho_{n0})\, v_{in}^0 t O(\alpha) (V_{s1}^*/c)^{5/3}.$$

Substitution of this expression in Eq. (16) yields

$$\rho_2 \cong \rho_0 \frac{\gamma + 1}{\gamma - 1} \frac{\frac{(\gamma + 1)^2}{6}\left[\frac{75\varkappa}{16\pi}(\gamma_p - 1) + \left(\frac{V_{s0}}{c}\right)^2 - \frac{15}{2}(\gamma_p - 1)\, v_{in}^0 t O(\alpha) \left(\frac{V_{s1}^*}{c}\right)^{5/3}\right]}{(\gamma_p - 1) A(\gamma) + (\gamma + 1)[2QB + (\Delta_T/r_2)\varphi_1 (V_{s0}/c)^2 (\gamma + 1)^2/2]}. \tag{27}$$

As is evident from Eq. (27), the amplitude of the density wave is determined to a significant degree by the electron component of the plasma: its thermal conductivity and electron temperatures T_{e0} in the absence of disturbance and T_{e2} behind the front. The amplitude ρ_2 depends on time. Initially because of energy pumping into the ion-sound wave its amplitude increases.

At

$$\frac{\Delta_T}{r_2}\left(\frac{V_{s0}}{c}\right)^2 > \frac{4}{(\gamma+1)^2}B, \quad \left(\frac{V_{s0}}{c}\right)^2 > \frac{75\varkappa}{16\pi}(\gamma_p - 1)$$

the amplitude reaches "saturation":

$$\rho_{2\max} \approx \rho_0 (r_2/\Delta_T)\,[(\gamma-1)\,\varphi_1]^{-1}.$$

However increases in amplitude are hindered by energy loss as a result of collisions with neutral particles ahead of the shock wave front, since the portion of the energy conveyed by the plasma component to neutrals increases with time as $t^{11/5}$, while the term with $(V_{s0}/c)^2$ increases as $t^{6/5}$. If $(V_{s0}/c)^2 < 1$, then ρ_2 depends only weakly on time and its value is close to that of Eq. (10); the difference from Eq. (10) being that some "attenuation coefficient" less than unity appears, which may be neglected in coarse estimates. Equation (27) was obtained for a strong shock wave $[q = (V_{s0}/c)^2(1 + T_{e0}/T_{i0})^{-1} \ll 1]$ using an upper bound estimate for Ω. This in principle limits its applicability.

In conclusion, we note that the simple form of Eqs. (7), (8), (18)-(22) was obtained with the assumption that the parameter

$$\alpha = \xi/r_2 = q^{1/6}\,(l/r_0)\,(V_s/a_0)^2\,(5/2)^{2/3}\sqrt{\gamma}\,\sqrt[3]{\gamma\varkappa}$$

is much less than unity (r_0 is the characteristic dynamic length of the point explosion problem). This implies smallness of the ratio of the spatial scale of the leading edge of the plasma disturbance ξ to the shock wave radius r_2.

Application. We will consider a supernova flare as a point explosion in the interstellar plasma. As is well known [1], supernova flares are one of the most important factors maintaining the motion of the interstellar medium, which on average is a weakly ionized plasma. We will describe the adiabatic stage of supernova envelope motion on the basis of point explosion theory. This stage begins approximately 10^3 years after the flare and is completed after 30-50 thousand years [10]. Then in the following stage of evolution over 10-15 thousand years the supernova envelope cools from 10^6 to 10^3 K basically because of intense scintillation of gas behind the shock wave front. It cannot be excluded that during the course of these two stages conditions may develop for formation of an ion-sound disturbance with the following parameters preceding the envelope: plasma component velocity ≈100 km/sec, with amplitude several times greater than the background in the undisturbed gas; spatial scale of the plasma component disturbance ahead of the front $\xi = V_S^2/(c\nu_{in}^0) \approx 10^{18}$ cm $[\nu_{in}^0 = 10^{-16}\,(cm^2)\sqrt{4k_BT_i/(\pi m_i)}N_n,\ T_i = 10^2$ K, $N_n = 0.6$ cm$^{-3}]$. For comparison, the wave front radius at the end of the scintillation stage is 10^{19} cm, while $r_0 > 10^{21}$ cm. These estimates are approximate, since ion viscosity was not considered in the original Eq. (2). On the other hand, the appearance of supersonic protons ahead of the wave front can lead to development of instability and the appearance of turbulent viscosity as the dominant energy dissipation mechanism.

LITERATURE CITED

1. V. G. Gorbatskii, Cosmic Gas Dynamics [in Russian], Nauka, Moscow (1977).
2. B. S. Punkevich and B. M. Stepanov, "Ionization of an undisturbed argon shock wave in detonation of an explosive charge," in: Physical Processes in Combustion and Explosion [in Russian], Atomizdat, Moscow (1980).
3. V. A. Gorshkov, A. I. Klimov, A. N. Koblov, et al., "Shock wave propagation in a glow discharge plasma in the presence of a magnetic field," Zh. Tekh. Fiz., 54, No. 5 (1984).
4. I. A. Klimishin, Shock Waves in Stellar Envelopes [in Russian], Nauka, Moscow (1984).
5. G. I. Maikapar (ed.), Non-Equilibrium Physicochemical Processes in Aerodynamics [in Russian], Mashinostroenie, Moscow (1972).
6. Ya. B. Zel'dovich and Yu. P. Raizer, Shock Wave Physics and High Temperature Hydrodynamic Phenomena [in Russian], Fizmatgiz, Moscow (1963).
7. A. L. Velikovich and M. A. Liberman, Shock Wave Physics in Gases and Plasma [in Russian], Nauka, Moscow (1987).
8. R. F. Avramenko, A. A. Rukhadze, and S. F. Teselkin, "Shock wave structure in a weakly ionized non-isothermal plasma," Pis'ma Zh. Éksp. Teor. Fiz., 34, No. 9 (1981).
9. L. I. Sedov, Similarity and Dimensionality Methods in Mechanics [in Russian], Nauka, Moscow (1987).
10. I. S. Shklovskii, "Supernovaflares and the interstellar medium," Astron. Zh., 39, 209 (1962).